Block Diagrams, Feedback and Transient Response Specifications

This module introduces the concepts of system block diagrams, feedback control and transient response specifications which are essential concepts for control design and analysis.

$with(inttrans)$:

(This command loads the functions required for computing Laplace and Inverse Laplace transforms. For more information on Laplace transforms, see the *Laplace Transforms and Transfer Functions* module.)

Block Diagrams and Feedback

Consider the example of a common household heating system. A household heating system usually consists of a thermostat that measures the room temperature and compares it with an input desired temperature. If the measured temperature is lower than the desired temperature, the thermostat sends a signal that opens the gas valve and starts the combustion in the furnace. The heat from the furnace is then transferred to the rooms of the house which causes the air temperature in the rooms to rise. Once the measured room temperature exceeds the desired temperature by a certain amount, the thermostat turns off the furnace and the cycle repeats. This is an example of a control system and in this case the variable being controlled is the room temperature. This system can be sub-divided into its major parts and represented by the following diagram which shows the directions of information flow. This type of diagram, known as a block diagram, is very useful in understanding how the different components interact and effect the variable of interest.

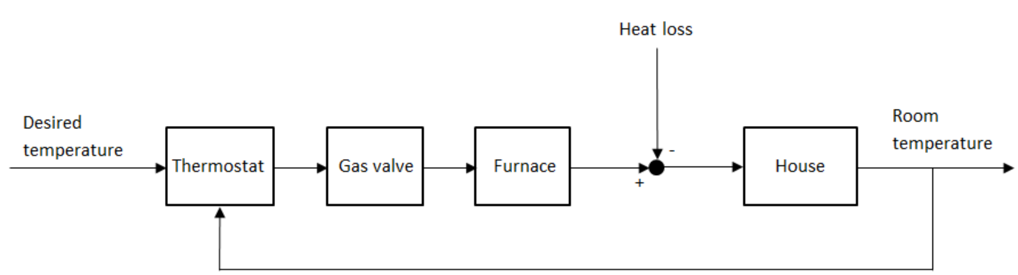


Fig. 1: Block diagram of a household heating system

The gas valve, furnace and house can be combined to get one block which can be called the *plant* of the system. In general, the plant is the aggregate part of a system that takes the control signal from the controller as an input and outputs the variable being controlled.

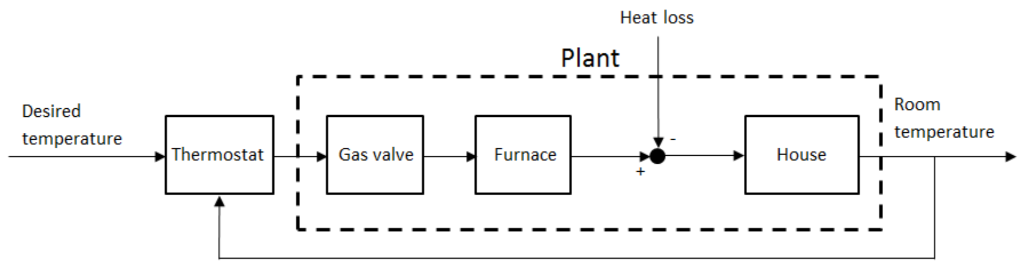


Fig. 2: System plant

Fig. 3 shows the general block diagram for a system with feedback control. *Feedback* refers to the returning of the measurements of a controlled variable to the controller so that it can be further used to influence the controlled variable.

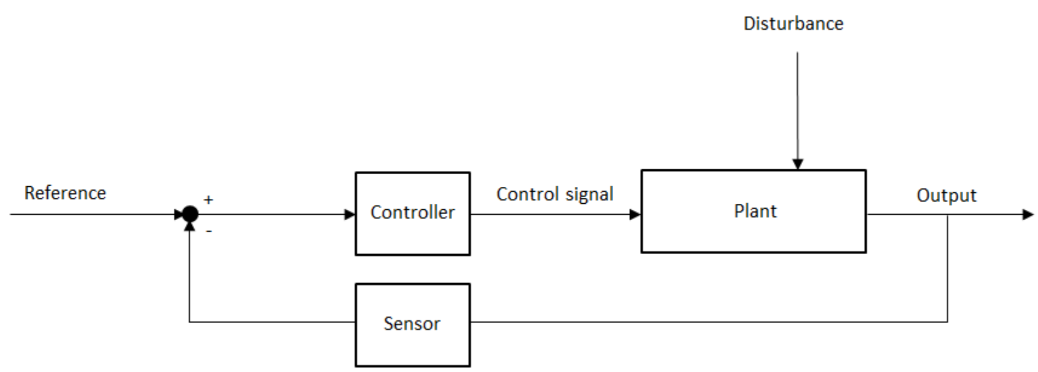


Fig. 3: Block diagram with feedback

If the system equations can be written such that its components only interact such that the output of one component is the input of another, then the system can be represented by a block diagram of transfer functions in the Laplace domain (see Fig. 4). This is a very useful tool because it facilitates in obtaining the equations of a system (including the effects of a controller) and studying its behavior.

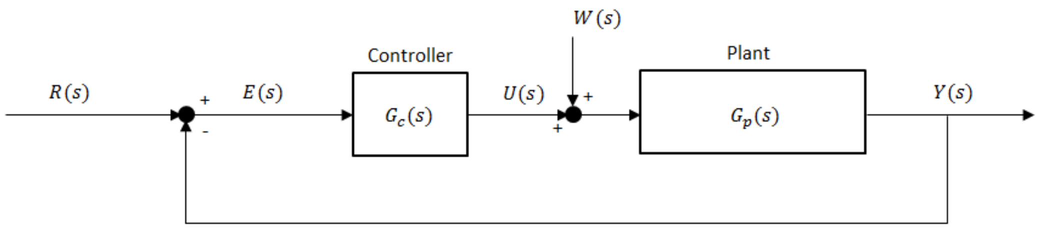


Fig. 4: Block diagram of transfer functions

In Fig. 4 $R(s)$ is the reference signal, $E(s)$ is the error signal, $G_c(s)$ is the controller transfer function, $U(s)$ is the control signal, $W(s)$ is the disturbance signal, $G_p(s)$ is the plant transfer function and $Y(s)$ is the output signal. From the definition of a transfer function (see the Laplace Transforms and Transfer Functions module) the following part of the diagram

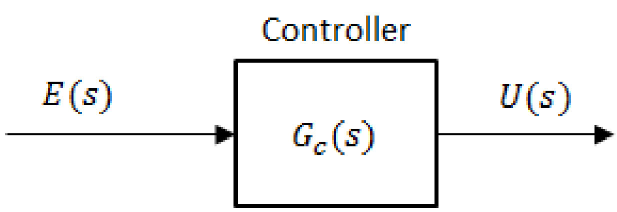


Fig. 5: The controller block

is equivalent to the equation

$$E(s) \cdot G_c(s) = U(s)$$

... Eq. (1)

In words, this means that, in the Laplace domain, the input signal multiplied by the transfer function gives the output signal. Similarly, the following part of the diagram

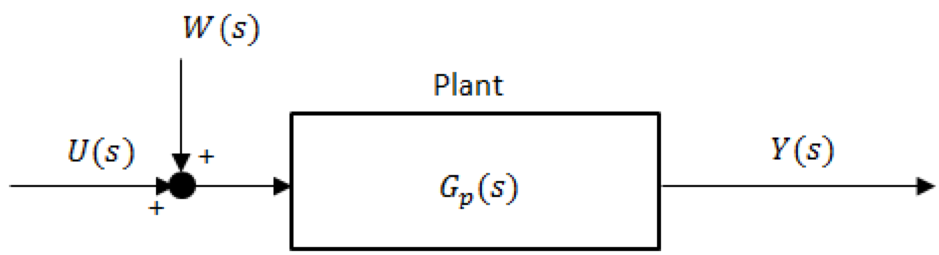


Fig. 6: The plant block

is equivalent to the equation

$$[U(s) + W(s)] \cdot G_p(s) = Y(s)$$

... Eq. (2)

Using Eqs. (1) and (2), the following equation for the relation between the error signal and the output signal can be found.

$$\left[E(s) \cdot G_c(s) + W(s)\right] \cdot G_p(s) = Y(s)$$

... Eq. (3)

This shows how the block diagram can be used to relate different inputs and outputs. This process is continued to obtain the relation between the reference signal $R(s)$ and the output $Y(s)$. The following part of the diagram

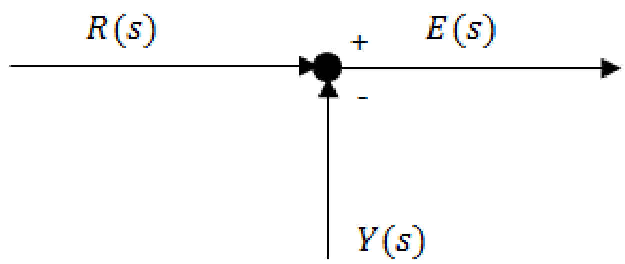


Fig. 7: Error signal

is equivalent to

$$R(s) - Y(s) = E(s)$$

... Eq. (4)

Eqs. (3) and (4) can be combined to get

$$\left[[R(s) - Y(s)] \cdot G_c(s) + W(s) \right] \cdot G_p(s) = Y(s)$$

... Eq. (5)

This equation can be rewritten as

$$R(s) \cdot G_c(s) \cdot G_p(s) + W(s) \cdot G_p(s) = Y(s) \cdot \left[1 + G_c(s) \cdot G_p(s) \right]$$

... Eq. (6)

or

$$Y(s) = R(s) \cdot \left[\frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)} \right] + W(s) \cdot \left[\frac{G_p(s)}{1 + G_c(s) \cdot G_p(s)} \right]$$

... Eq. (7)

If there is no disturbance ($W(s) = 0$), then

$$Y(s) = R(s) \cdot \left[\frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)} \right]$$

... Eq. (8)

and the equivalent system diagram is

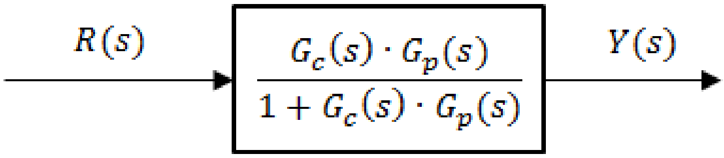


Fig. 8: Equivalent system diagram

Here

$$\frac{G_c(s) \cdot G_p(s)}{1 + G_c(s) \cdot G_p(s)}$$

... Eq. (9)

is called the closed loop transfer function. This is because it includes the effects of the feedback loop. From this it can be concluded that if the plant transfer function $G_p(s)$ and the controller transfer function $G_c(s)$ are known, then the reference input can be multiplied to the closed loop transfer function to obtain the output of the system (the inverse Laplace transform of this would give the time response).

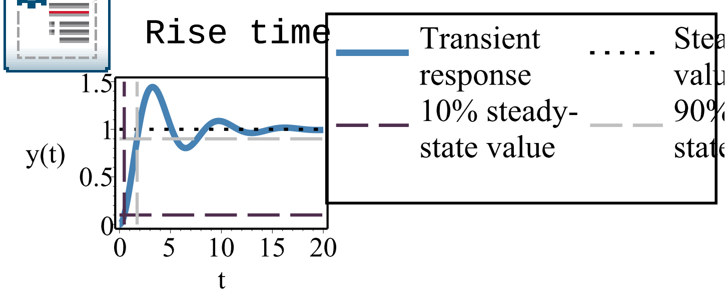
Transient Response Specifications

The step response of a system is an important performance characteristic because it determines how quickly and accurately a system responds to changing inputs. The following specifications are commonly used to define the performance/ requirements of a system's step response.

Definitions

Rise time

The rise time t_r is defined as the time it takes the transient response to move from 10% to 90% of the steady state response.



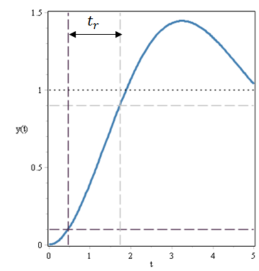


Fig. 9: Rise time

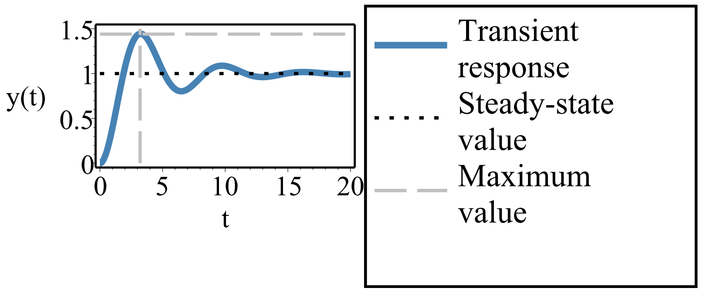
Maximum Overshoot

The maximum overshoot M_p is the percentage by which the maximum value $y_{\max}$ of the transient response exceeds the steady state value y_{ss}.

$$M_p = \frac{|y_{\max} - y_{ss}|}{|y_{ss}|} \cdot 100\ \%$$

... Eq. (10)

Maximum overshoot



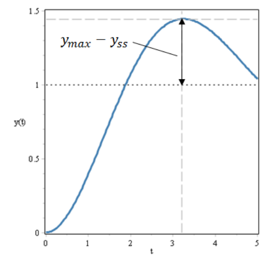
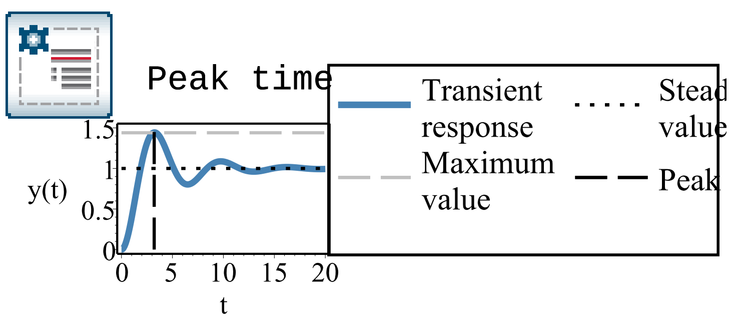
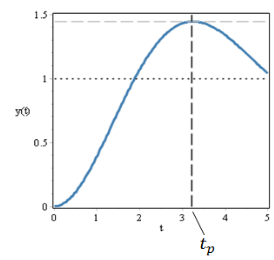


Fig. 10: Maximum overshoot

Peak Time

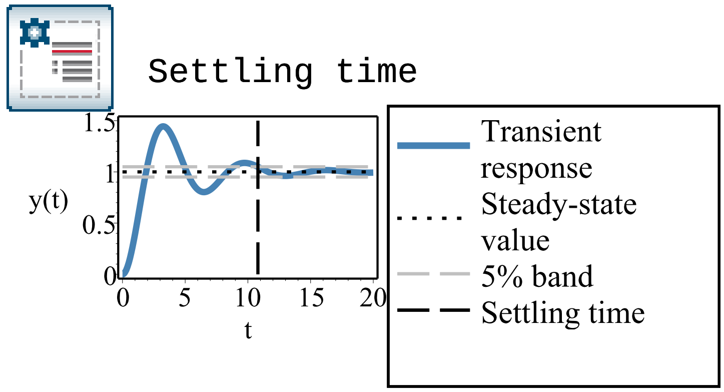
The peak time t_p is defined as the time at which the maximum overshoot occurs.





Settling Time

The settling time is defined as the time after which the output is within a specified band around the steady state value. The specified band is usually ±1% or ±5% of the steady state value.



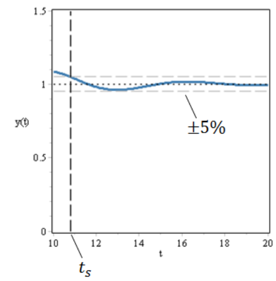


Fig. 11: Settling time

First Order Systems

The following is the characteristic form of a first order system.

... Eq. (11)

Here $y(t)$ is the controlled variable, $u(t)$ is the input and K and a are constants.

In the Laplace domain, the characteristic form is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K \cdot a}{s + a}$$

which can also be written as

... Eq. (12)

by replacing $\tau = \frac{1}{a}$.

DC Gain, *K*

The constant K is called the DC gain of the system and is defined as the ratio of the magnitude of the steady-state output of the system to the input. For a unit-step input ($Y(s) = \frac{1}{s}$),

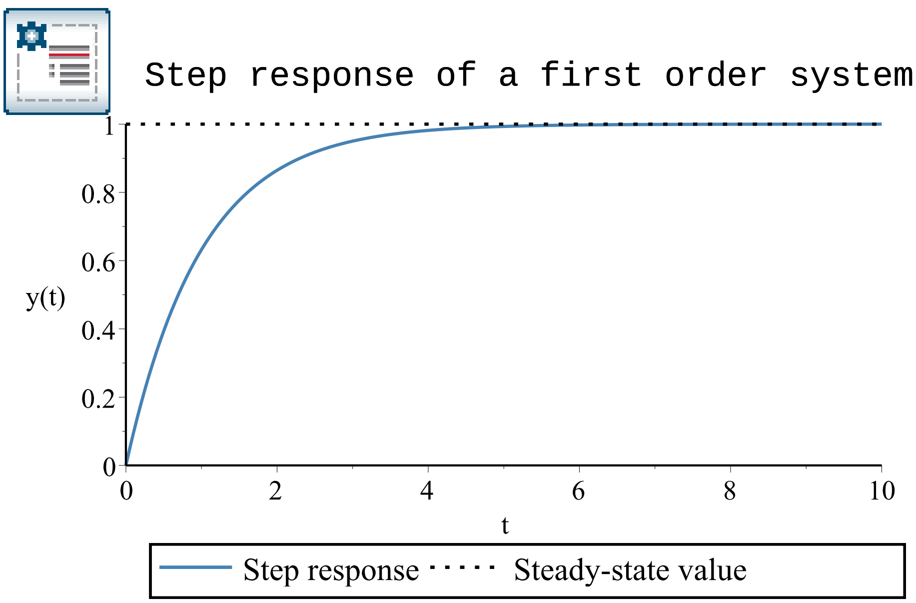
$$y_{ss} = \lim_{t \to \infty} y(t)$$

$$= \mathcal{L}^{-1}\left[\lim_{s \to 0} s \cdot \frac{1}{s} \cdot G(s) \right]$$

$$= \mathcal{L}^{-1}\left[\lim_{s \to 0} \frac{K}{\tau \cdot s + 1} \right]$$

$$= \mathcal{L}^{-1}[K] = K$$

If, $K = 1$ and $\tau = 1$, the following plot shows the system response to a step input.



Time Constant, τ

The parameter τ is called the time constant of the system and is a measure of the speed of the response. The step response of a first order system is

$with(inttrans)$:

$$invlaplace\left(\frac{1}{s} \cdot \frac{K}{\tau \cdot s + 1}, s, t\right)$$

$$K\left(-e^{-\frac{t}{\tau}}+1\right) \qquad \textbf{(2.2.2.1)}$$

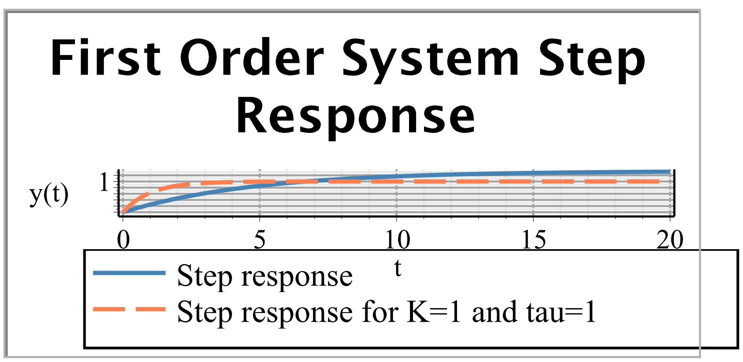
and the slope at $t = 0$ is

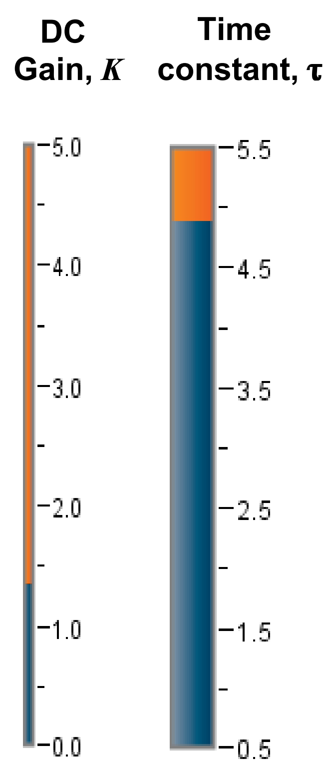
$$eval\left(\frac{d}{d\,t}\,\textbf{(2.2.2.1)},\ t=0\right)$$

$$\frac{K}{\tau} \qquad \textbf{(2.2.2.2)}$$

A larger time constant means that the system response takes a longer time to rise.

The time constant and DC gain of a first order system can be varied using the gauges (below) to see the effect on the unit-step response of the system.





Response Specifications for a First Order System

As shown above, the time response of a first order system to a step-input is

$$y(t) = K \cdot \left(-e^{-\frac{t}{\tau}} + 1 \right)$$

... Eq. (13)

Rearranging for t,

$$t = -\tau \cdot \ln\left(1 - \frac{y(t)}{K}\right)$$

... Eq. (14)

This equation can be used to find equations for the rise time t_d and the settling time t_s. The maximum overshoot M_p and peak time t_p are not defined for a first order step response because there is no overshoot and hence no peak.

Rise time

The rise time is defined as the time it takes for $y(t)$ to go from $0.1 \cdot K$ to $0.9 \cdot K$,

$$t_d := -\tau \cdot \ln\left(1 - \frac{0.9 \cdot K}{K}\right) - \left(-\tau \cdot \ln\left(1 - \frac{0.1 \cdot K}{K}\right)\right) :$$

$$t_d = 2.197\ \tau$$

... Eq. (15)

Settling time

The settling time for a $\pm 5\%$ band is

$$t_s := -\tau \cdot \ln\left(1 - \frac{0.95 \cdot K}{K}\right) :$$

$$t_s = 2.996\ \tau$$

... Eq. (16)

Second Order Systems

The characteristic form of a second order system is

$$\ddot{y}(t) + 2\cdot\zeta\cdot\omega_n\cdot s\cdot\dot{y}(t) + \omega_n^{2}\cdot y = K\cdot\omega_n^{2}\cdot u(t)$$

... Eq. (17)

In the Laplace domain, this can be written as

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K\cdot\omega_n^{2}}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{2}}$$

... Eq. (18)

Here, ζ is called the damping ratio, ω_n is called the natural frequency and K is the DC gain. (A second order system is analogous to a spring-mass system with damping. See the *Intro. to Vibration* modules for more information). The poles of this transfer function (for $0 < \zeta < 1$) are

$$s = -\zeta\cdot\omega_n \pm \mathrm{j}\cdot\omega_n\cdot\sqrt{1-\zeta^2}$$

... Eq. (19)

This shows that the system will have an oscillatory response to step and impulse inputs.

Assuming that $0 < \zeta < 1$, the impulse response ($U(s) = 1$) is

$$y(t) = \mathcal{L}^{-1}[G(s)] = K\cdot\left(\frac{\omega_n^{2}}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\cdot\omega_n\cdot t}\cdot\sin\left(\sqrt{1-\zeta^2}\cdot\omega_n\cdot t\right)\right)$$

... Eq. (20)

And the step response ($U(s) = \frac{1}{s}$) is

$$y(t) = \mathcal{L}^{-1}\left[\frac{1}{s}\cdot G(s)\right] = K\left(1 - \frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\zeta\cdot\omega_n\cdot t}\cdot \sin\left(\sqrt{1-\zeta^2}\cdot\omega_n\cdot t + \theta\right)\right)$$

... Eq. (21)

where $\theta = \cos^{-1}(\zeta)$. Since the frequency of the oscillations is $\sqrt{1-\zeta^2}\cdot\omega_n$, this is also known as damped frequency ω_d. Fig. (12) illustrates the relation between the parameters and the pole locations.

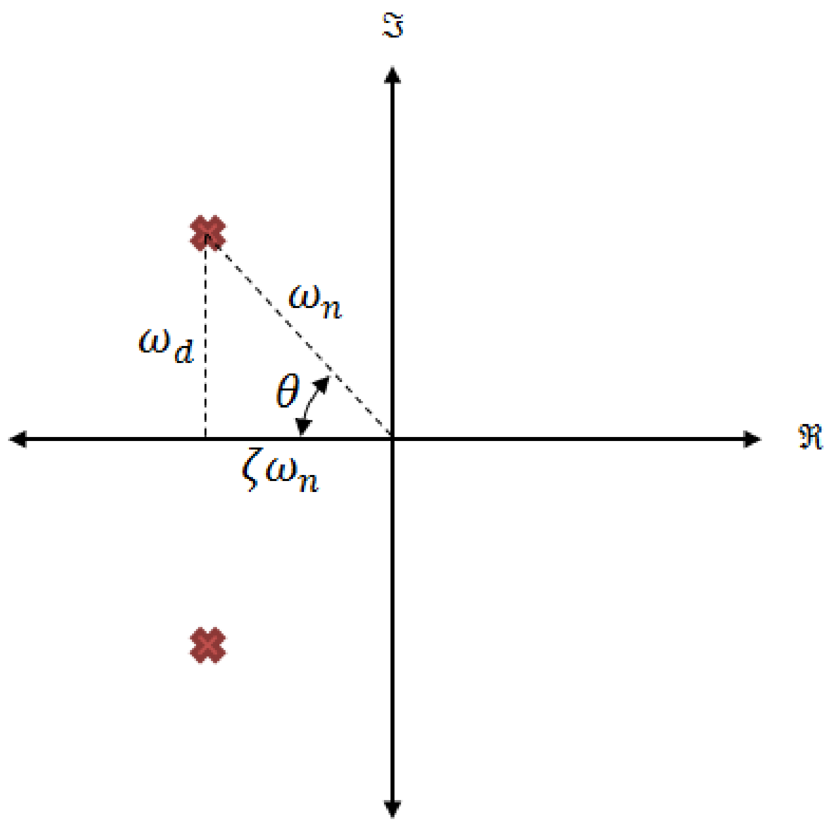


Fig: 12: Pole locations of a second order system

If $\zeta > 1$, then the system responses will have different forms than the ones shown above (without any sinusoidal terms). In most dynamic systems of interest (there are many exceptions) the damping ratio is not higher than 1 which is why the response of a system with $0 < \zeta < 1$ is of more interest.

Response Specifications for a Second Order System

Rise time

Due to the lack of a simple analytical equation for the rise time of a second order system, the following are some of the approximations that are commonly used (with increasing level of accuracy).

$$t_r \cong 1.8 \cdot \omega_n$$

... Eq. (22)

$$t_r \cong \frac{0.8 + 2.5 \cdot \zeta}{\omega_n}$$

... Eq. (23)

$$t_r \cong \frac{1 - 0.4167 \cdot \zeta + 2.917 \cdot \zeta^2}{\omega_n}$$

... Eq. (24)

Settling time

The following is an approximation for the settling time with a band of ±5%.

$$t_s \cong \begin{cases} \dfrac{3.2}{\zeta \cdot \omega_n} & 0 < \zeta \le 0.69 \\ \dfrac{4.5 \cdot \zeta}{\omega_n} & 0.69 < \zeta < 1 \end{cases}$$

... Eq. (25)

Peak time and maximum overshoot

It is easier to find an analytical expression for the maximum overshoot. The slope of the response curve at the point of maximum overshoot is zero (the first derivative is zero). In the Laplace domain, taking a derivative is equivalent to multiplying by s. So for a step response, the impulse response is the slope and is equal to zero at the peak time.

Equating the impulse response to zero gives

$$K \cdot \left(\frac{\omega_n^{\,2}}{\sqrt{1-\zeta^2}} \cdot e^{-\zeta \cdot \omega_n \cdot t} \cdot \sin\left(\sqrt{1-\zeta^2} \cdot \omega_n \cdot t \right) \right) = 0$$

... Eq. (26)

which occurs when

$$\sin\left(\sqrt{1-\zeta^2} \cdot \omega_n \cdot t \right) = 0$$

... Eq. (27)

which implies that

$$t = \frac{n \cdot \pi}{\sqrt{1-\zeta^2} \cdot \omega_n}$$

... Eq. (28)

Since the maximum over shoot happens the first time that the slope is zero,

$$t_p = \frac{\pi}{\sqrt{1-\zeta^2}\cdot\omega_n}$$

... Eq. (29)

This can be substituted into Eq.(21) to get

$$y(t_p) = K\cdot\left(1 - \frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\frac{\zeta\cdot\omega_n\cdot\pi}{\sqrt{1-\zeta^2}\cdot\omega_n}}\cdot\sin\left(\frac{\sqrt{1-\zeta^2}\cdot\omega_n\cdot\pi}{\sqrt{1-\zeta^2}\cdot\omega_n} + \cos^{-1}(\zeta)\right)\right)$$

$$= K\cdot\left(1 - \frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\frac{\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}\cdot\sin\left(\pi + \cos^{-1}(\zeta)\right)\right)$$

$$= K\cdot\left(1 + \frac{1}{\sqrt{1-\zeta^2}}\cdot e^{-\frac{\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}\cdot\sqrt{1-\zeta^2}\right)$$

$$= K\cdot\left(1 + e^{-\frac{\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}\right)$$

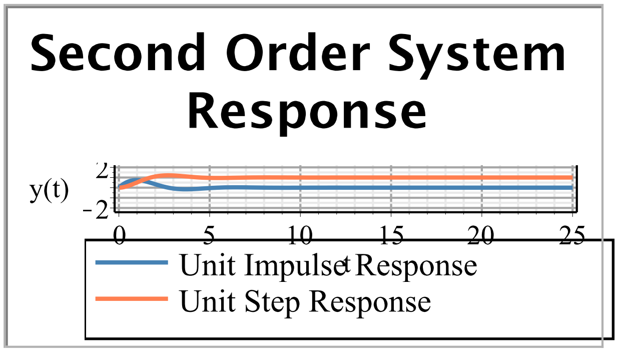
... Eq. (30)

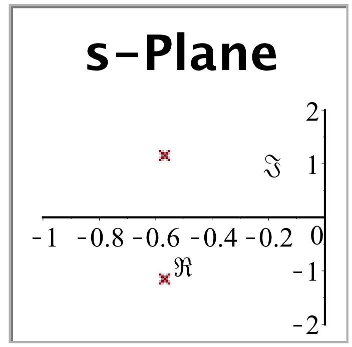
Therefore, the maximum overshoot is

$$M_p = e^{-\frac{\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}\ \%$$

... Eq. (31)

For the following plot, the complex plane (below right) can be used to change the locations of the complex poles of a second order system (with $K = 1$) and see the effect on the system specifications. (Click somewhere on the complex plane or drag the poles)





Damping ratio	Natural frequency	Damped frequency
$\zeta =$ 0.4417	$\omega_n =$ 1.287 rad/sec	$\omega_d =$ 1.155 rad/sec

Rise time	Settling time (5% band)	Peak time	Maximum overshoot
$t_r =$ 0.8364 sec	$t_s =$ 5.628 sec	$t_p =$ 2.722 sec	$M_p =$ 21.29 %

Example 1

restart;

Problem Statement: The transfer function of a plant is $G_p(s) = \frac{18}{s^2 + 3 \cdot s + 9}$.

Find:

a) the rise time t_r

b) the 5% settling time t_s

c) the percent overshoot M_p

d) and the peak time t_p

Solution:

To begin, the natural frequency and damping ratio of the system need to be found. Comparing the transfer function to the form of Eq. (18), the natural frequency is

$\omega_n := 3:$

and the damping ratio is

$\textbf{local}\ \zeta := \frac{3}{2 \cdot \omega_n}$

$$\frac{1}{2} \tag{3.1}$$

(Here ζ is defined locally because it is a protected name in Maple)

These two parameters can be used to find the specifications:

Part a) The rise time

Using Eq. (24), the rise time (in seconds) is

$t_r := \frac{1 - 0.4167 \cdot \zeta + 2.917 \cdot \zeta^2}{\omega_n}$

$$0.5069666667 \tag{3.1.1}$$

Part b) The settling time

Using Eq. (25), the 5% settling time (in seconds) is

$t_s := \frac{3.2}{\zeta \cdot \omega_n}$

$$2.133333333 \tag{3.2.1}$$

Part c) The percent overshoot

Using Eq. (31), the percent overshoot is

$M_p := e^{-\frac{\zeta \cdot \pi}{\sqrt{1-\zeta^2}}} \cdot 100$

$$100\, e^{-\frac{1}{3} \pi \sqrt{3}} \tag{3.3.1}$$

$\xrightarrow{\text{at 5 digits}}$

$$16.303 \qquad (3.3.2)$$

Part d) The peak time

Using Eq. (29), the peak time (in seconds) is

$$t_p := \frac{\pi}{\sqrt{1-\zeta^2}\cdot\omega_n}$$

$$\frac{2}{9}\pi\sqrt{3} \qquad (3.4.1)$$

$$\xrightarrow{\text{at 5 digits}} \quad 1.2092 \qquad (3.4.2)$$

Example 2

Problems Statement: A proportional controller is added to the system of Example 1, as shown in the following block diagram.

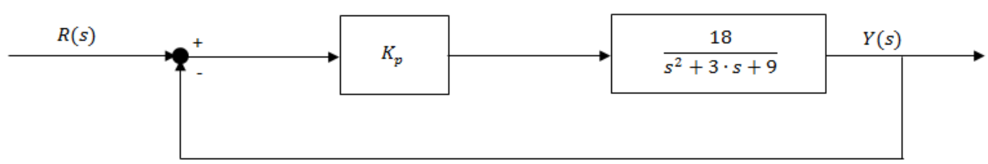


Fig. 13: System transfer function with proportional control

What should the value of K_p (proportional control gain) be to reduce the percent overshoot to 10%? Plot the step response for the system with and without the proportional control after determining the required K_p value.

Solution:

The closed loop transfer function in this case is

$$\frac{K_p \cdot G_p(s)}{1 + K_p \cdot G_p(s)} = \frac{K_p \cdot 18}{s^2 + 3 \cdot s + 9 + K_p \cdot 18}$$

First, Eq. (31) is used to find the value of the required damping ratio ζ_{req}:

$$10 = e^{-\frac{\zeta_{req} \cdot \pi}{\sqrt{1 - \zeta_{req}^2}}} \cdot 100$$

$$10 = 100\, e^{-\frac{\zeta_{req}\,\pi}{\sqrt{-\zeta_{req}^2 + 1}}} \quad \textbf{(4.1)}$$

$solve(\textbf{(4.1)}, \zeta_{req})$

$$\frac{\ln(10)}{\sqrt{\pi^2 + \ln(10)^2}} \quad \textbf{(4.2)}$$

$\xrightarrow{\text{at 5 digits}}$

$$0.59115 \quad \textbf{(4.3)}$$

Therefore, the required damping ratio is

$\zeta_{req} := \textbf{(4.3)}$

$$0.59115 \quad \textbf{(4.4)}$$

Comparing the closed loop transfer function to Eq. (18),

$3 = 2 \cdot \zeta_{req} \cdot \omega_{n\ req}$

$$3 = 1.18230\, \omega_{n\ req} \quad \textbf{(4.5)}$$

$solve(\textbf{(4.5)}, \omega_{n\ req})$

$$2.537427049 \quad \textbf{(4.6)}$$

Therefore, the required natural frequency is

$\omega_{n\ req} := \textbf{(4.6)}$

$$2.537427049 \quad \textbf{(4.7)}$$

Once again, comparing the closed loop transfer function to Eq. (18),

$\omega_{n\ req} = \sqrt{9 + 18 \cdot K_p}$

$$2.537427049 = 3\sqrt{1 + 2\, K_p} \quad \textbf{(4.8)}$$

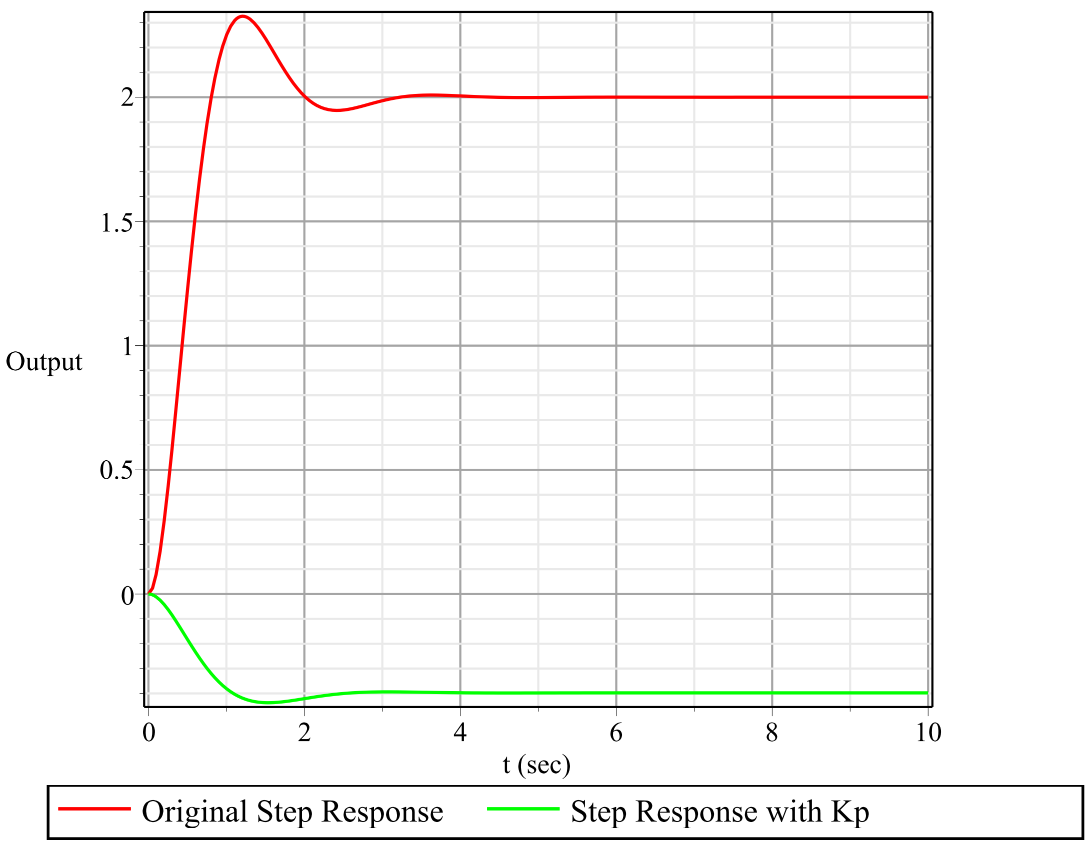
$K_p := solve(\textbf{(4.8)}, K_p)$

$$-0.1423035539 \quad \textbf{(4.9)}$$

Therefore, the gain of the proportional controller has to be negative to reduce the percentage overshoot. However, this also pushes the steady state response of the system farther away from the input value and makes it negative.

$with(inttrans);$

$$plot\left(\left[invlaplace\left(\frac{1}{s}\cdot\frac{18}{s^2+3\cdot s+9}, s, t\right), invlaplace\left(\frac{1}{s}\cdot\frac{K_p\cdot 18}{s^2+3\cdot s+9+K_p\cdot 18}, s, t\right)\right], t=0..10,\right.$$
$$color=[red, green], axes=boxed, gridlines=true, labels=["\text{t (sec)}", "\text{Output}"], legend$$
$$\left.=["\text{Original Step Response   }", "\text{Step Response with Kp}"]\right);$$



From this it can be seen that even though the percent overshoot decreased, the steady-state output of the system is farther away from the input value than before.

Example 3 (with MapleSim)

Problem Statement: A Proportional-Derivative (PD) controller is a type of controller that reacts proportionally to the error and rate of change of error. A PD controller is added to the transfer function of Example 1 as shown in the following diagram.

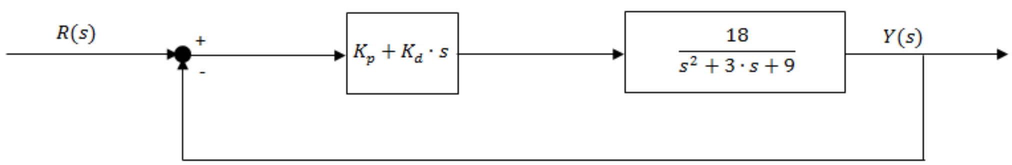


Fig. 14: System transfer function with PD control

Find the values of K_p and K_d so that the steady-state error is 0.1 and the percent overshoot is 10% (for a step input).

Solution:

With Maple

restart;

The closed loop transfer function is

$$\frac{(K_p + K_d \cdot s) \cdot G_p(s)}{1 + (K_p + K_d \cdot s) \cdot G_p(s)} = \frac{18\,(K_p + K_d \cdot s)}{s^2 + (3 + 18 \cdot K_d) \cdot s + (9 + 18 \cdot K_p)}$$

Comparing this transfer function with Eq. (18) the steady state error is

$$e_{ss} = 1 - \lim_{s \to 0} s \cdot \left(\frac{1}{s} \cdot \frac{18\,(K_p + K_d \cdot s)}{s^2 + (3 + 18 \cdot K_d) \cdot s + (9 + 18 \cdot K_p)} \right)$$

$$e_{ss} = 1 - \frac{2\,K_p}{1 + 2\,K_p} \qquad \textbf{(5.1.1)}$$

This can be be used to find the required K_p value. For a steady state error of 0.1,

$$K_p := solve\left(0.1 = 1 - \frac{2\,K_p}{2\,K_p + 1},\, K_p\right)$$

$$4.50 \qquad \textbf{(5.1.2)}$$

Assuming that this transfer function can be treated as a second order system by ignoring the effect of the zero, the natural frequency of the system is

$$\omega_n := \sqrt{9 + 18 \cdot K_p}$$

$$9.487 \qquad \textbf{(5.1.3)}$$

and the damping ratio is

$$\zeta_{req} := solve\left(10 = \mathrm{e}^{-\frac{\zeta_{req} \cdot \pi}{\sqrt{1 - \zeta_{req}^{2}}}} \cdot 100,\, \zeta_{req}\right)$$

$$\frac{\ln(10)}{\sqrt{\pi^2 + \ln(10)^2}} \qquad \textbf{(5.1.4)}$$

$\xrightarrow{\text{at 5 digits}}$

$$0.59115 \qquad \textbf{(5.1.5)}$$

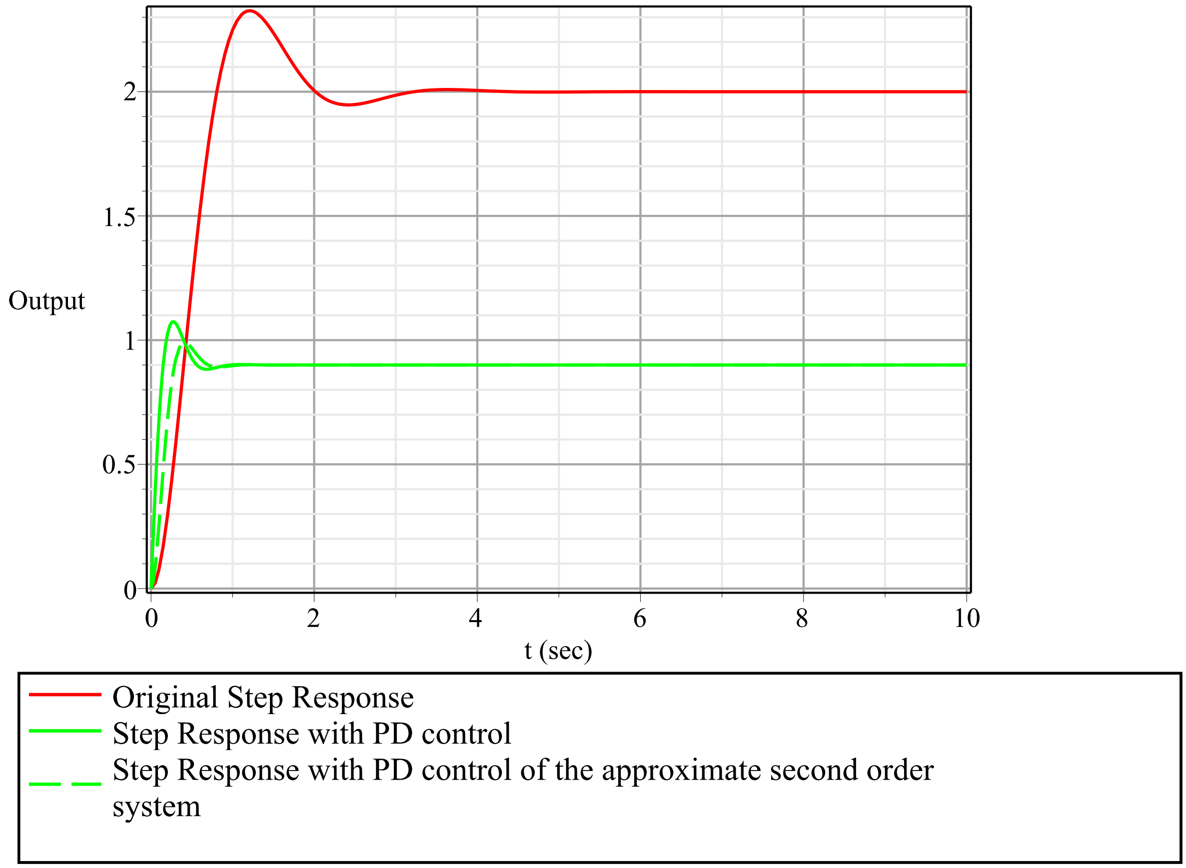
These two parameters can be used to calculate the required K_d value.

$$K_d := solve\left(2 \cdot \zeta_{req} \cdot \omega_n = 3 + 18 \cdot K_d,\, K_d\right)$$

$$.456 \qquad \textbf{(5.1.6)}$$

The following plot shows the original step response without a controller, the step response with a PD controller and the calculated controller gains, and the step response with the approximate second order system used to calculate the controller gains.

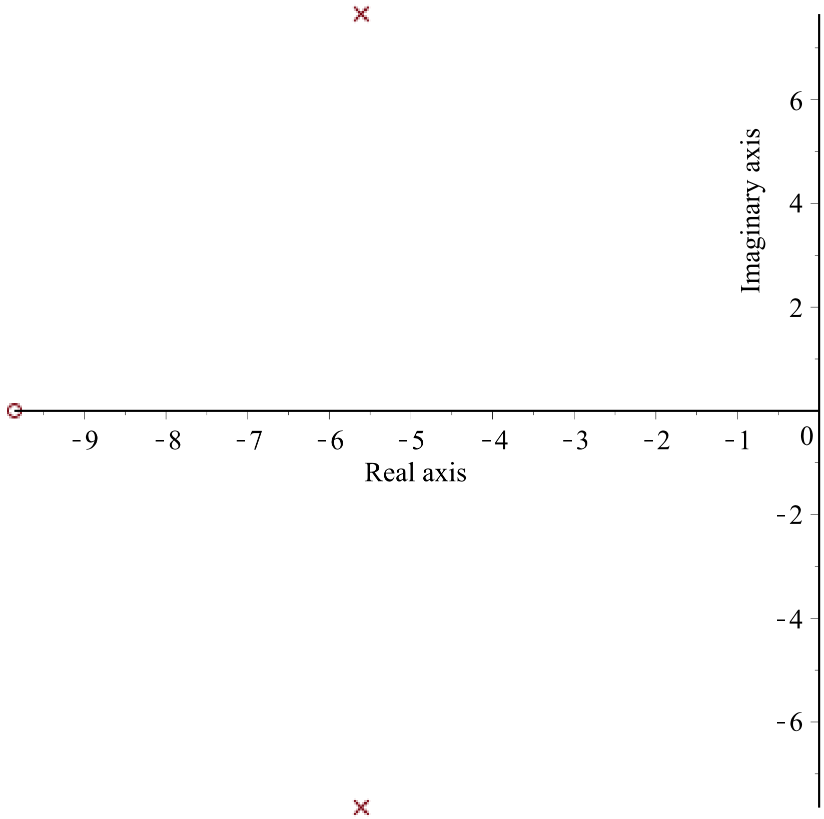
Plot



The specifications are not exactly met due to the zero, as can be seen from the plot. A negative zero located on the real axis is equivalent to adding the impulse response of the second order system. So the overshoot in this case is greater than the required overshoot. However, since the zero is relatively large in magnitude (z≈-10), its effect decays very quickly. The following plot shows the pole and zero locations.

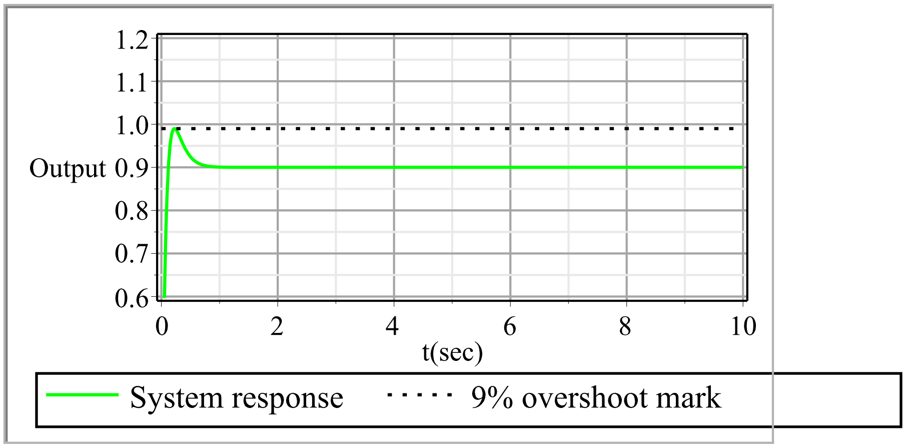
with(*DynamicSystems*) :

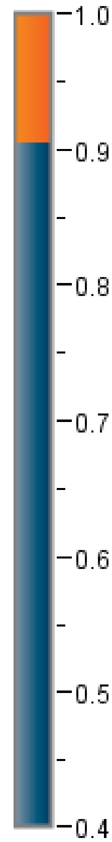
$$ZeroPolePlot\left(TransferFunction\left(\frac{18\left(K_p + K_d \cdot s\right)}{s^2 + \left(3 + 18 \cdot K_d\right) \cdot s + \left(9 + 18 \cdot K_p\right)}\right), labels = ["\text{Real axis}", "\text{Imaginary axis}"]\right)$$



The following plot and gauge can be used to fine tune the controller to get the required K_d value (approximately 0.9 for this case).

K_d





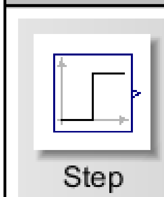
With MapleSim

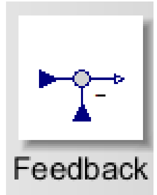
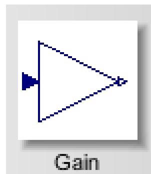
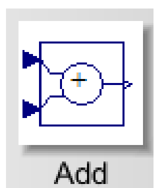
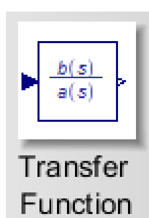
Constructing the Model

Step 1: Insert Components

Drag the following components into the workspace:

Table 1: Components and locations

Component	Location
 Step	Signal Blocks > Common
	Signal Blocks > Common

	
 (2 required)	Signal Blocks > Common
	Signal Blocks > Mathematical > Functions
	Signal Blocks > Common
	Signal Blocks > Continuous

Step 2: Connect the components

Connect the components as shown in the following diagram:

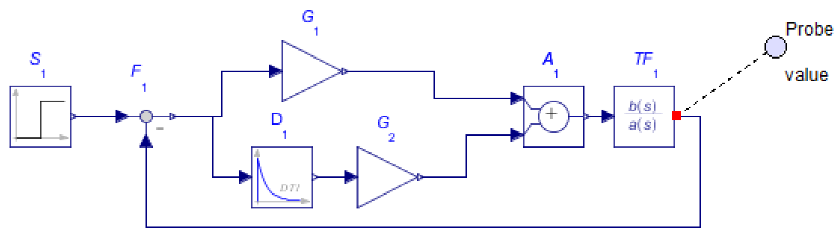


Fig. 15: Component diagram

Step 3: Set up the components

1. Click the proportional **Gain** component (G_1 in the diagram) and enter a value for the gain **(*k*).**
2. Click the derivative **Gain** component (G_2 in the diagram) and enter a value for the gain **(*k*).**
3. Click the **Transfer Function** component and enter **[18]** for ***b*** and **[1, 3, 9]** for ***a***.

The gain values that satisfy the required specifications can be found using a systematic trial and error approach. The approximate values calculated in the previous subsection can be used as a starting point. To better understand the behavior of the system, one of the various other types of input signals (available under **Sources** in the component library) can also be used instead of the step input.

Simulation of an equivalent physical model

In the problem statement, the transfer function of the system is provided without any details of the physical system that it represents. There are many possible systems that this second order transfer function could represent. For example, it could represent a machine component modeled as a spring mass system with viscous damping and the following parameters:

Table 2: Spring-mass system parameters

Parameters	Value
Mass, m	$\frac{1}{18}$ kg
Spring	

constant, k	$\frac{1}{2}$ N/m
Damping coefficient, b	$\frac{1}{6}$ N·s/m

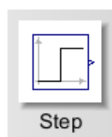
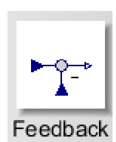
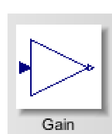
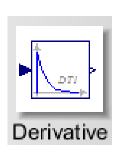
This subsection shows how this problem can be simulated if this physical system was specified instead of the transfer function.

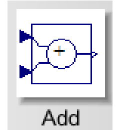
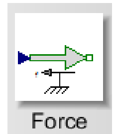
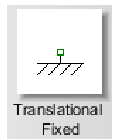
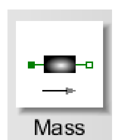
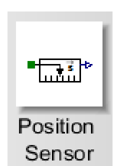
Constructing the model

Step1: Insert Components

Drag the following components into the workspace:

Table 3: Components and locations

Component	Location
 Step	Signal Blocks > Common
 Feedback	Signal Blocks > Common
 Gain (2 required)	Signal Blocks > Common
 Derivative	Signal Blocks > Mathematical > Functions

 Add	Signal Blocks > Common
 Force	1-D Mechanical > Translational > Force Drivers
 Translational Fixed	1-D Mechanical > Translational > Common
 Translational Spring Damper	1-D Mechanical > Translational > Common
 Mass	1-D Mechanical > Translational > Common
 Position Sensor	1-D Mechanical > Translational > Sensors

Step 2: Connect the components

Connect the components as shown in the following diagram (the dashed boxes are not part of the model, they have been drawn on top to help make it clear what the different components are for).

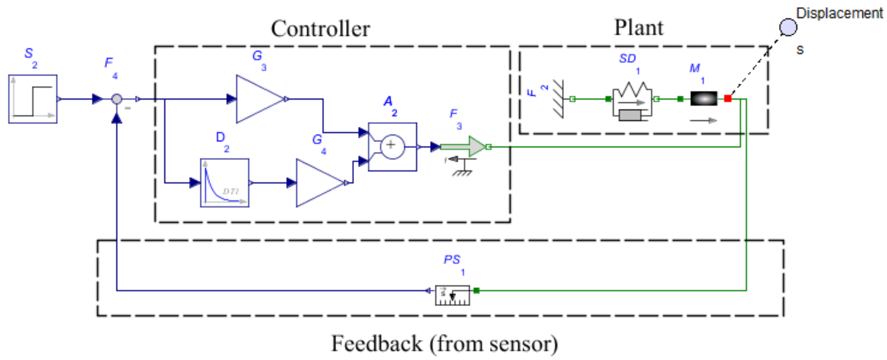


Fig. 16: Component diagram

Step 3: Set parameters and initial conditions

1. Click the proportional **Gain** component and enter a value for the gain **(*k*).**
2. Click the derivative **Gain** component and enter a value for the gain **(*k*).**
3. Click the **Translational Spring Damper** component and enter $\frac{1}{2}$ *N/m* for the spring constant **(*c*)** and $\frac{1}{6}$ *N/m* for the damping constant **(*d*)**.
4. Click the **Mass** component and enter $\frac{1}{18}$ *kg* for the mass **(*m*)**, **0** *m/s* for the initial velocity **(v_0)** and **0** *m* for the initial position **(s_0)**. Select the check marks that enforce these initial condition.

Step 4: Run the Simulation

1. Attach a **Probe** to the **Mass** component as shown in Fig. 16. Click this probe and select **Length** in the **Inspector** tab. This will show the position of the mass as a function of time.
2. Click **Run Simulation** ( ).

This simulation will provide the same results as the simulation using the transfer function.

Reference:

G.F. Franklin et al. "Feedback Control of Dynamic Systems", 5th Edition. Upper Saddle River, NJ, 2006, Pearson Education, Inc.